Problem 16

Is it true that $f \circ (g + h) = f \circ g + f \circ h$.

Solution

It's not true that $f \circ (g + h) = f \circ g + f \circ h$. Take, for example,

$$f(x) = \sqrt{x}$$

$$g(x) = \cos x$$

$$h(x) = \sin x.$$

Then

$$\begin{aligned} f \circ (g + h) &= f(g(x) + h(x)) \\ &= \sqrt{\cos x + \sin x} \\ &\neq \sqrt{\cos x} + \sqrt{\sin x} \\ &= f(g(x)) + f(h(x)) \\ &= f \circ g + f \circ h. \end{aligned}$$